The Factors Influencing the Numerical Simulation of the Spectral Analysis of the Surface Waves Method

Pei-Hsun Tsai [1]   Chang-hai Chien[2]*   Zheng-yi Feng[2]   Yu-ting Lin[2]

ABSTRACT   A numerical study was performed to identify the significant factors that influence numerical simulation of the spectral analysis of the surface waves (SASW) method. Damping, impulsive force magnitude, time step, element size of mesh, and soil layer profile were all parametrically studied. An axisymmetric finite difference code was adopted to simulate the surface wave propagation in layered media. The phase spectra method was utilized to obtain the dispersion curves and shear wave velocity profiles of the soil strata. The most important factor influencing the SASW numerical modeling is the shear wave velocity profile of the soil layers. The shear wave velocity of the soil layer also dominates the fluctuation of the cross power spectrum.

Key Words : Spectral analysis of surface waves (SASW), cross power spectrum, shear wave velocity, numerical simulation.

1. Introduction

The surface wave method provides an efficient tool to evaluate the dynamic properties of media. The spectral analysis of surface waves method (SASW) is a non-destructive testing method proposed by Nazarian et al. (1983). This method uses the theory of surface wave propagation to measure the dynamic properties of soil media. Through the signal processing, the plot of shear wave velocity versus soil depth can be obtained.

Although the SASW is a simple and convenient method, the SASW still has its shortcomings. For example, when it is used in complex terrain, computational errors could results in signal processing due to reflection, refraction and interference of Rayleigh waves (Sheu et al., 1988; Hiltunen and Woods, 1990).

The SASW collects the phase information of the cross power spectrum by changing the distance between source and the nearest receiver and the spacing of receivers. The phase velocity versus frequency or wavelength is plotted to obtain the dispersion curve. Then, a complete plot of shear wave velocity versus soil depth is obtained. However, some factors can affect the phase information. One of these factors is the layout of a survey line. Due to the effects of the near and far field, the phase angle information is processed according to filtering criteria pro-

posed by some researchers. There are several researches using numerical simulation for the SASW to understand the influence of layout of survey line for dispersion curves. Gucunski and Woods (1992) used a numerical method to simulate surface waves in layered soil media with different soil layer thickness. They proposed to use Heisey's filtering criteria (Heisey et al., 1982) to reduce the influence of near and far fields. Chen (2006) used the software, Fast Lagrangian Analysis of Continua (FLAC, Itasca, 2008), to investigate the influence of noise signals on dispersion curves. Lin and Lin (2007) used the two-dimensional finite difference method to simulate surface waves in a lateral heterogeneous medium. Their results showed that the "high-lateral-resolution" surface wave method could reduce the influence of the lateral heterogeneity of the medium and improve the spatial resolution. Karray and Lefebvre (2009) used FLAC to simulate a surface wave and captured time-domain data to investigate the techniques of multiple filters and time variable filters on Rayleigh wave mode separation.

Lin and Feng (2010) used the Hilbert-Huang transform (HHT) and low-mode filter (impulse response windowing filtering, IRF) to process the SASW signals. The original signal was processed with IRF and empirical mode decomposition (EMD) to investigate the dispersion curves of regular and irregular profiles.

Although the cross power spectrum method to obtain dispersion curves and thus velocity profiles are not new and is only a traditional method, we do not focus on the data reduction methodology for the SASW. In this paper, we focused on the influence of damping, impulsive force magnitude, time step, mesh size, and soil layer profile on the SASW test by numerical simulation and the reasonable ranges of the factors were discussed. The dispersion curves were obtained by cross power spectrum (hereafter called the "phase spectrum").

2. The SASW Method

1. Dispersion curve and the shear wave velocity profile of soil layers

The fundamental theories of the SASW method are described in detail in the literature, such as Nazarian et al. (1983), Jiang (2004), Lin and Feng (2010), Lin (2010), etc. We illustrate the analysis procedure of the dispersion curve and the shear wave velocity profile as follows:

Performing the SASW test requires setting up two receivers in a survey line on the ground surface. An impulsive source by hammer or falling weight is applied at the end point of the survey line to generate shock waves. The vertical velocity histories at the two receivers are recorded.

In this study, seismic data obtained from different receiver spacings were superimposed to establish a dispersion curve. The dispersion curves are used to deduce the shear wave velocity profile of the midpoint of the two receivers under different receiver spacings. Jiang (2004)

adopted the statistical method used by Nazarian and Stokoe in the mid-1980s to deduce a more reliable dispersion curve. He proposed an improved method to calculate the average dispersion curve and used the Satoh method (Satoh et al., 1991) to calculate the shear wave velocity profile. In this study, the MATLAB code developed by Jiang (2004) was used to obtain the dispersion curves and the shear wave velocity profile of soil layers. A schematic diagram of the calculation is shown in Fig. 1.

2. Construction of the numerical model for the SASW Method

We employed FLAC to simulate the SASW test and obtain the velocity time history of receivers. FLAC is an explicit finite difference code and suitable for the axisymmetrical dynamic problem. The soil layers are assumed to be homogeneous and isotropic elastic materials. The procedure of the numerical simulation and the required FLAC parameter settings are illustrated as follows.

(1) Declaration of the numerical model: axisymmetrical and dynamic configurations were used in this study.

(2) Generate mesh: The mesh used in this study was an axisymmetric grid that was 90 m in radius and 40 m in depth. The axisymmetrical axis was set at the left side of the mesh. Square elements were used.

(3) Assign material parameters of soil layers: The density, shear modulus (G), and bulk modulus of soils were assigned in FLAC analysis.

(4) Setting the boundary conditions: A quiet boundary was assigned to the right side and the bottom of the mesh to prevent wave reflection.
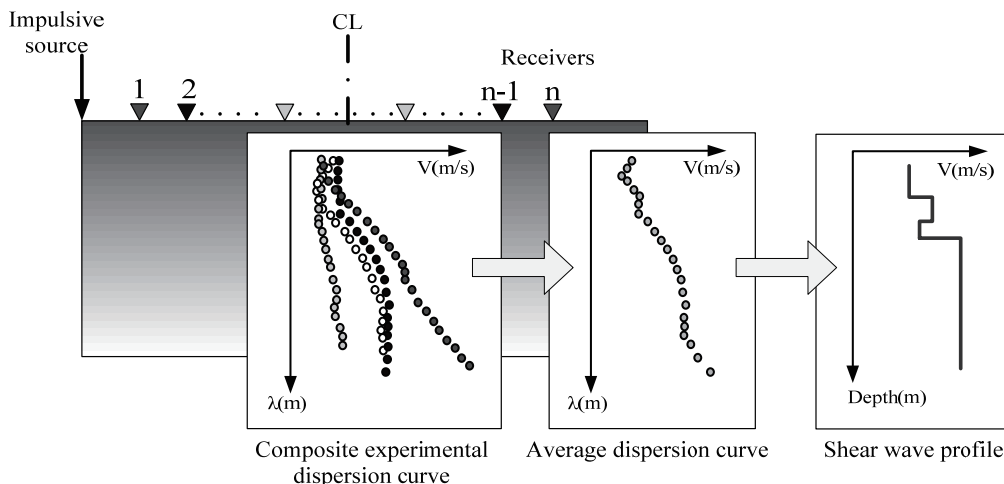

Fig.1 Schematic diagram of the calculation of dispersion curves and the shear velocity profile (Note: λ stands for surface wave wavelength)

(5) Assignment of Rayleigh damping: The fraction of the critical damping ratio and "center frequency" were input. The center frequency could be assumed to be the resonant frequency obtained by performing a fast Fourier transform (FFT) for the dynamic response of the soil system from a preliminary SASW simulation.

(6) Input of the impulsive force: A half-sine function was used to simulate the waveform of the SASW impulsive force. The duration of the applied impulsive force was set to 0.01 seconds.

(7) Perform dynamic calculations and record the velocity time history of receivers: The total time for the dynamic simulation was 2 seconds. The time step ($\triangle$t) directly affects the maximum sampling frequency. An effective choice of time step is important for the SASW modeling.

(8) The gravity field is ignored: Gravity in the SASW numerical model is not considered because we assumed this dynamic system is a linearly elastic system.

3. Parametric study

A numerical parametric study with various conditions of the SASW test was performed to discuss the influences of the parameters on the cross power spectrum. The simulation parameters are shown in Table 1. Boldface letters denote the baseline case of this study.

(1) Grid: The grid size was set to 0.25, 0.5, and 1 m for the parametric study. Kuhlemeyer and Lysmer (1973) proposed that the grid size (element size) must be smaller than 1/10 to 1/8 of the minimum wavelength corresponding to the maximum frequency when wave propagating at a fixed velocity. We also know that if the grid size is too small, the number of elements will increase drastically. Therefore, excessive computer memory and computing time will be consumed.

(2) Velocity of soil layer: The velocity is set to 50, 100, 200, 400, and 600 m/s. Other elastic soil parameters are shown in Table 2.

(3) Rayleigh damping ratio: The critical Rayleigh damping ratio of 1%, 2%, and 5% were purposely used in the parametric analysis to investigate the influence of damping.

(4) Time step ($\triangle$t): Two timesteps of 10^{-4} and 10^{-3} seconds were designated for comparison.

(5) Source magnitude: The magnitudes of the impulse

sources were 100, 500, and 1,000 N for comparison.

(6) Stratigraphy of soil profile: Assumption of the "regular" profile that the shear wave velocity increases with depth and the "irregular" layering were simulated. The thicknesses of soil layers were changed to investigate the variations of the phase spectrum. The five simulated cases of four regular and one irregular soil profiles are shown in Fig. 2.

4. Receiver locations

We used the distance between the source and the second receiver as twice the offset from the nearest station, as recommended by Heisey et al. (1982). The four different spacings of receivers of 4, 8, 16 and 24 m were assumed. As shown in Fig. 3, the four receiver pairs are S1-S2, S2-S3, S3-S5, and S4-S6. The corresponding dispersion curves of these four receiver pairs were obtained. The four dispersion curves were superimposed to obtain a composite dispersion curve and the plot of the shear wave velocity versus depth.

Table 1   Parameter variations in this study

Parameters	Variation of the parameters
Grid size (m)	0.25 **0.5** 1
Shear wave velocity of homogeneous single soil layer (m/s)	50 **100** 200 400 600
Rayleigh damping ratio (%)	**1** 2 5
Time step ($\triangle$t, s)	$1E^{-3}$ **$1E^{-4}$**
Impulsive force Magnitude (N) (Duration 0.01 second)	100 500 **1000**
Profile of soil layers	**Homogeneous single layer profile**; two-layer regular profile; three-layer regular profile; two-layer irregular profile

Note: The boldface letters denote the baseline case of this study.

Table 2   The assumed material properties of the simulated soil layers

	Mass density, ρ (kg/m^3)	Poisson's ratio, ν	Shear modulus, G (N/m^2)	Bulk modulus, K (N/m^2)	Young's modulus, E (N/m^2)
50	2000	0.33	$5.0E^6$	$1.30E^7$	$1.33E^7$
100	2000	0.33	$2.0E^7$	$5.22E^7$	$5.32E^7$
200	2000	0.33	$8.0E^7$	$2.09E^8$	$2.13E^8$
400	2000	0.33	$3.2E^8$	$8.35E^8$	$8.51E^8$
600	2000	0.33	$7.2E^8$	$1.88E^9$	$1.92E^9$

5. "Demand bandwidth" and "analyzable bandwidth" of the cross power spectrum

The effect of various parameters on the cross power spectrum was investigated in this study. To facilitate the discussion of the numerical simulation results, the "demand bandwidth" and "analyzable bandwidth" of the cross power spectrum are defined below.

(1) "Demand bandwidth" of the cross power spectrum

In this study, the shear wave velocity profiles were mostly calculated up to 24 m in depth. Therefore, we define the "demand bandwidth" as the frequency range of the cross power spectrum that is required to obtain the shear wave velocity profile corresponding to a depth from 1 m to 24 m. We also defined an "upper limit" frequency that is the maximum frequency required to obtain the shear wave velocity at 1 m depth.

(2) "Analyzable bandwidth" of the cross power spectrum

The "analyzable bandwidth" is defined in this paper as the frequency range of the cross power spectrum that is nomally wrapped with less noise and fluctuation, so that we can use that frequency range to calculate the corresponding phase velocity.

The "demand bandwidth" and "analyzable bandwidth" of the cross power spectrum are helpful to discuss the numerical results in this paper. However, there is no physical meaning for the two definitions.

3. Results and discussion

1. Effect of shear wave velocity of soil layer on the "demand bandwidth" of the cross power spectrum

The numerical simulations were performed with the parameters (Table 1) of the baseline case and the designated cases. The shear wave velocity of the soil layer was varied from 50 to 600 m/s. For soil shear wave velocities of 200 to 600 m/s, the grid size was set as 0.25 m.

The cross power spectrum and shear wave velocity were calculated using receiver spacings of 4 and 24 m. The computational errors of the calculated shear wave velocity were ranging from 5 to 15%. However, we found that the faster the shear wave velocity of the soil layer is, the smoother the wrapped cross power spectrum in the high-frequency range is. Therefore, we concluded that the

demand bandwidth of the phase angle increases as the shear wave velocity of the soil layer increases.

Fig. 4 shows the plot of the shear wave velocity of the soil layer versus the "upper limit of frequency", the maximum frequency required to obtain the velocity at 1 m depth, of the demand bandwidths. Fig. 4 shows almost a linear relationship between the velocities of the soil layers and the upper limits of the demand bandwidths. Thus, upper limits of frequency corresponding to various velocities can be estimated from Fig. 4. If the receiver spacing is smaller than 4 m, the upper limits of the demand bandwidths can be higher.

2. Effect of grid size

The effect of the grid sizes on the cross power spectrum was investigated by changing the grid size (0.25, 0.5, and 1 m) of the baseline case. The phase angle jumps caused by the grid size is discussed. The finer the grid size is, the larger the analyzable bandwidth of phase spectrum is.
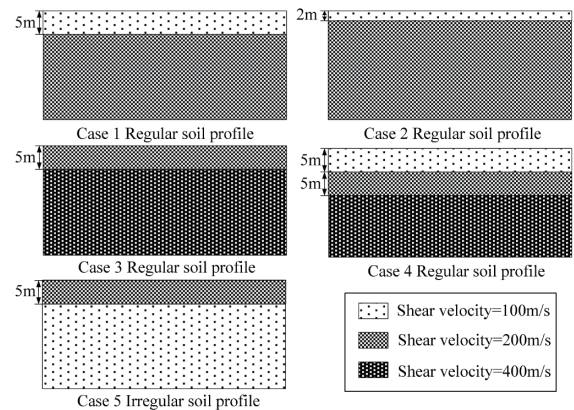
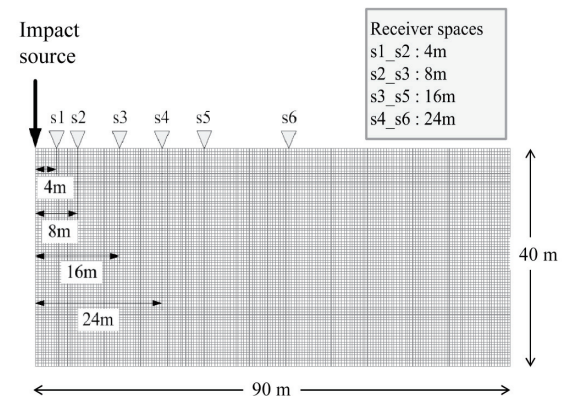**Fig.2   The simulated soil profiles of Case 1 to Case 5**

Fig.3   Locations of the receivers (velocity sensors) used in this study

When the grid size is 1 m (Fig. 5), the cross power spectrum of the 4-m receiver spacing is jagged above 30 Hz. The cross power spectrum of the 24 m receiver spacing also shows non-smooth wrapping. Fig. 4 shows that the upper limit of the demand bandwidth (corresponding to the shear wave velocity of soil layer Vs= 100 m/s) is 50 Hz. Therefore, the jagging above 30 Hz of the 1 m size mesh is not satisfactory for the computation of a soil layer with a 100 m/s of shear wave velocity. Fig. 6 shows the composited dispersion curves calculated from each receiver pair with different spacing. In Fig. 6, the dispersion curves of mesh sizes of 0.25 and 0.5 m were satisfactory; however, the dispersion curve of 1 m mesh size case is "ill-conditioned", with dramatic decreases in the phase velocity at the high frequency range, especially at $30 \sim 40$ Hz. The results of the 1 m mesh case cause an underestimation of the shear wave velocity at depths shallower than 2 m.

Therefore, the analyzable bandwidth of cross power spectrum using a grid size of 1 m is insufficient to analyze a soil layer with shear wave velocity of 100 m/s when soil depth is shallower than 2 m. A grid size that is smaller than 0.5 m is considered as satisfactory for this soil layer (Fig. 6).

The frequency of abnormal phase fluctuation corresponding to different grid sizes was parametrically analyzed. At some specific frequencies, abnormal phase fluctuation occurs for different grid sizes. The fluctuation of cross power spectrum appears at a lower frequency for a coarser grid, and thus the analyzable bandwidth is smaller. Therefore, if a larger analyzable bandwidth is needed, the grid size used should be smaller.

This parametrical study also showed that the phase fluctuation is related to the shear wave velocity of soil layer and the spacing of receivers. For a higher wave velocity of soil layer, the receiver spacing should not be too large to avoid the influence of phase fluctuations on the demand bandwidth of the cross power spectrum. The shear wave velocities of soil layers and the corresponding demand bandwidths of cross power spectrum are shown in Table 3. This table can serve as a reference for choosing a mesh size for numerical simulations of the SASW tests.

3. Effect of time step (△t)

The parametric study of time step used △t of 10^{-3} seconds and 10^{-4} seconds for the baseline case. That is, the maximum sampling frequencies were 1,000 Hz and 10,000 Hz, respectively. The grid size was changed to

0.25 m for comparison reasons because the corresponding analyzable bandwidth of 0.25 m grid size is larger than that of a grid size of 0.5 m.

Table 3　Demand bandwidth versus shear wave velocity of soil

Shear wave velocity of soil (m/s)	Demand bandwidth (Hz)	Suitable mesh size (m)
50	$0 \sim 25$	$\dfrac{\lambda_{min}}{8} \sim \dfrac{\lambda_{min}}{2}$
100	$0 \sim 50$	$\dfrac{\lambda_{min}}{8} \sim \dfrac{\lambda_{min}}{4}$
200	$0 \sim 100$	$\dfrac{\lambda_{min}}{8} \sim \dfrac{\lambda_{min}}{4}$
400	$0 \sim 200$	$\dfrac{\lambda_{min}}{8}$
600	$0 \sim 300$	$\dfrac{\lambda_{min}}{8}$

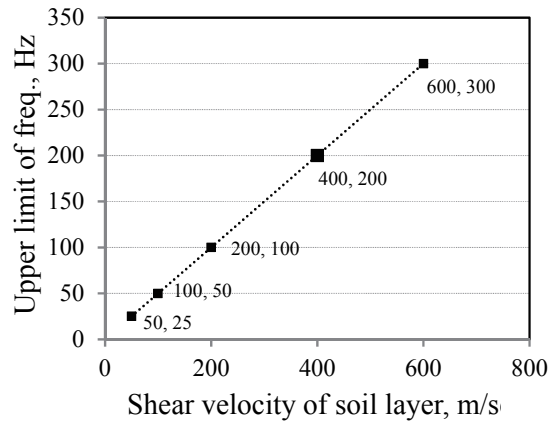Note that λ is the wavelength of the lowest velocity of the soil layers.

Fig.4　Shear wave velocities of soil layers versus the upper limit frequency of the demand bandwidth of the phase spectrum for a receiver spacing of 4 m
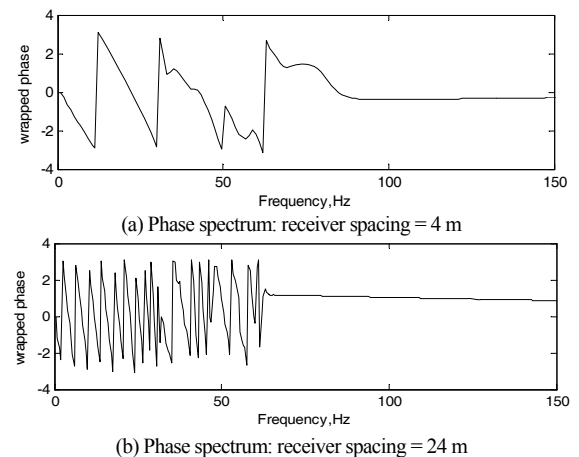

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m

Fig.5　Phase spectra obtained by using 1-m grid size

For the two time steps, it was found that the cross power spectra were almost identical for frequencies lower than 150 Hz. The cross power spectra show differences when the frequency is higher than 150 Hz. This frequency, 150 Hz, is higher than the demand bandwidth of 50 Hz (Table 3). Therefore, we consider that these two time steps are both satisfied for the numerical simulations.

4. Effect of Rayleigh damping ratio

The influence of Rayleigh damping ratios of 1, 2, and 5% were compared based on the baseline case. The analyzable bandwidths of cross power spectra for various spacings of receivers were compared. We found that the analyzable bandwidth decreased as the Rayleigh damping ratio increased. In particular, it decreased rapidly in the case of larger receiver spacings (24 m). When the Rayleigh damping ratio was high, the reduction of abnormal phase fluctuation by using a smaller mesh size is limited. This is due to that the stress waves decaying quite fast in the high damping materials.

5. Effect of the impact source magnitude

Impact source magnitudes of 100, 500, and 1000 N were used to investigate their influences on the numerical simulation of the SASW test. The displacement on ground surface became smaller as the source magnitude was smaller. However, the phenomenon of wave propagation was the same. That is, the shear wave velocities obtained by the SASW method are the same in these cases. If the damping of soils is higher or shear wave velocity in deeper soil is required, a larger impact source is needed to induce a larger impact source magnitude.

6. Effect of soil profile

The relationships of soil profiles (Fig. 2) and the demand bandwidth of cross power spectrum were investigated. The soil profiles studied included regular and irregular profiles. The regular profile is a multilayer media where the shear wave velocity increases with increasing depth. The irregular profile is a low-velocity layer or layers covered with a hard layer with high velocity.

  (1) Regular profile

    ① Case 1: Two layers (the depth of the upper layer and the lower layer are 5 and 35 m, and the shear wave velocities of the layers are 100 and 200 m/s, respectively).

      From Fig. 7, it can be seen that cross power spectrum between 60 and 90 Hz has irregular
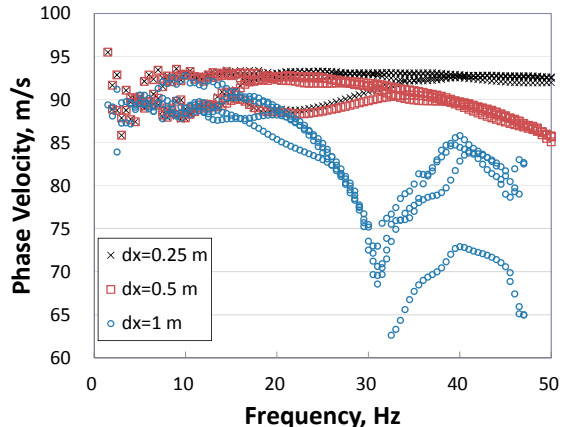

Fig.6 Combined dispersion curves for varied sensor distances of 0.25, 0.5, and 1-m grid sizes

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m
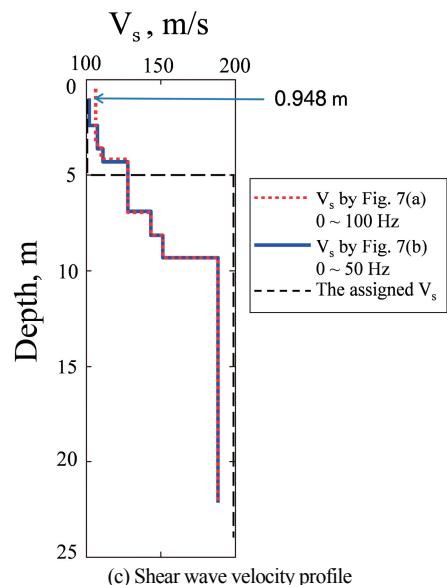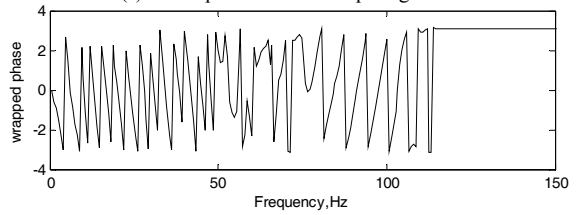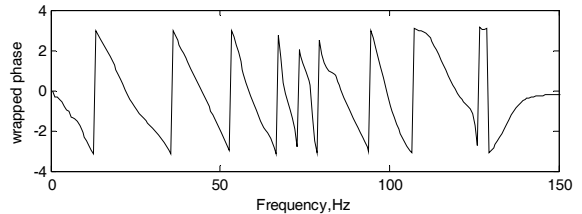

(c) Shear wave velocity profile

Fig.7 Phase spectra and shear wave velocity profile obtained in Case 1

wrapping for the 4 m receiver spacing case. For the 24 m receiver spacing case, the cross power spectrum is well wrapped only when the frequencies are lower than 50 Hz. Taking the frequencies lower than 50 Hz of the cross power spectra for receiver spacings of 1 and 24 m, the shear wave velocity can be obtained at depths as shallow as 1 m. Therefore, the frequencies lower than 50 Hz is defined as the "demand bandwidth" of the cross power spectrum for this soil profile. The upper limit of demand bandwidth is 50 Hz.

② Case 2: Two layers (the depth of the upper layer and the lower layer are 2 and 38 m, and the shear wave velocities of soil are 100 and 200 m/s, respectively).

For the 4 m receiver spacing case, the analyzable bandwidth is lower than 70 Hz. For the 24 m receiver spacing case, the analyzable bandwidth is lower than 40 Hz (Fig. 8). From the shear wave velocity profile obtained, the analyzable bandwidth lower than 50 Hz using 4 m receiver spacing is sufficient to calculate the shear wave velocity for the shallowest depth of 1 m.

③ Case 3: Two layers (the depth of the upper layer and the lower layer are 5 and 35 m, and the shear wave velocities of soil are 200 and 400 m/s, respectively).
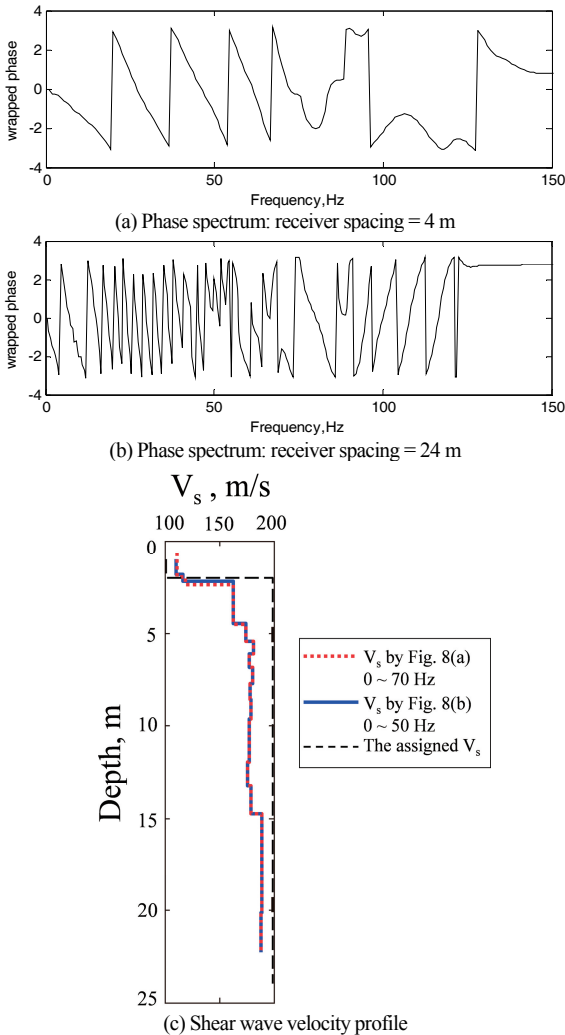
From the cross power spectrum (Fig. 9) for the 4 m receiver spacing case, the analyzable bandwidth is lower than 100 Hz; for the 24 m
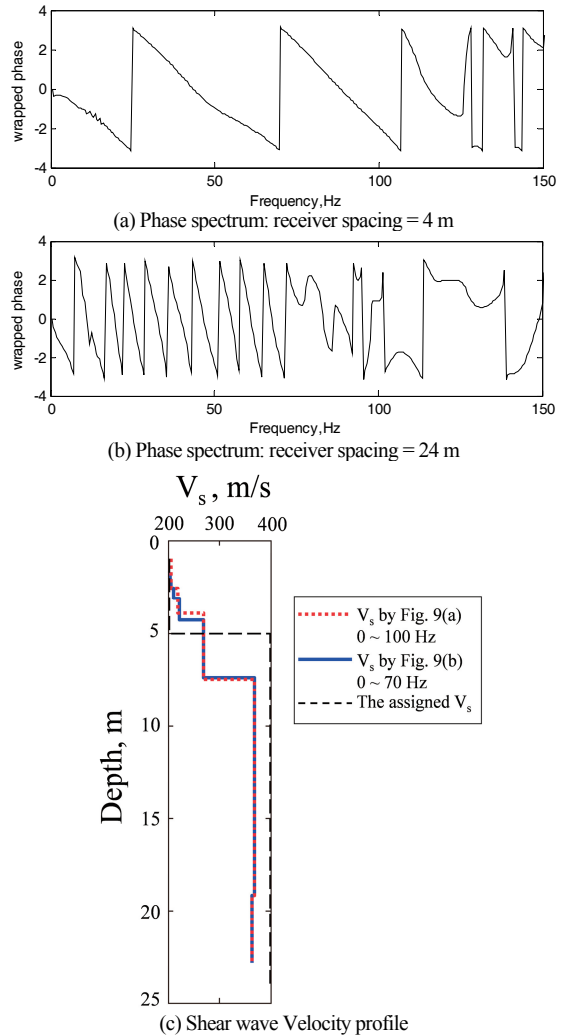

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m

(c) Shear wave velocity profile

Fig.8　Phase spectra and shear wave velocity profile obtained in Case 2

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m

(c) Shear wave Velocity profile

Fig.9　Phase spectra and shear wave velocity profile obtained in Case 3

receiver spacing case, the analyzable bandwidth is lower than 70 Hz. If a bandwidth is lower than 70 Hz and 4 m receiver spacing are used to calculate the shear wave velocity profile, then the calculated shallowest depth of shear wave velocity is about 2 m, which does not fulfill the requirement of 1 m. If the bandwidth lower than 100 Hz is used for wave velocity calculation, the shallowest depth calculated is 1 m. Therefore, the demand bandwidth is 100 Hz for this case.

④ Case 4: Three layers (the depth of the upper, middle, and the lower layers are 5, 5, and 30 m, and the shear wave velocities of soil are 100, 200, and 400 m/s, respectively).
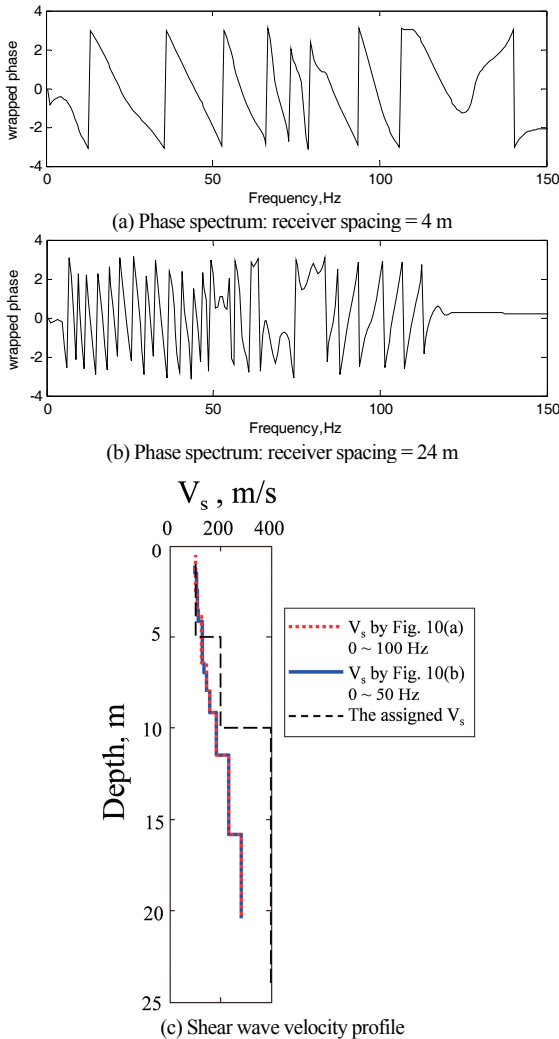
From the cross power spectrum (Fig. 10), for the 4 m receiver spacing case, the analyzable bandwidth is lower than 100 Hz. For the 24 m receiver spacing case, the available phase angle bandwidth is lower than 50 Hz. In Case 4, the demand bandwidth of 0 ~ 50 Hz is sufficient to obtain the shear wave velocity at 1 m.

In addition, from the shear wave velocities (Fig. 10), there are large discrepancies between the calculated and assumed wave velocities at depths deeper than 10 m. The calculated shear wave velocity was significantly underestimated. In cross power spectrum for the two receiver spacings, the wrapping of cross power spectra at low frequency is poorer than those of the other cases. The quality of the spectrum at low fre-
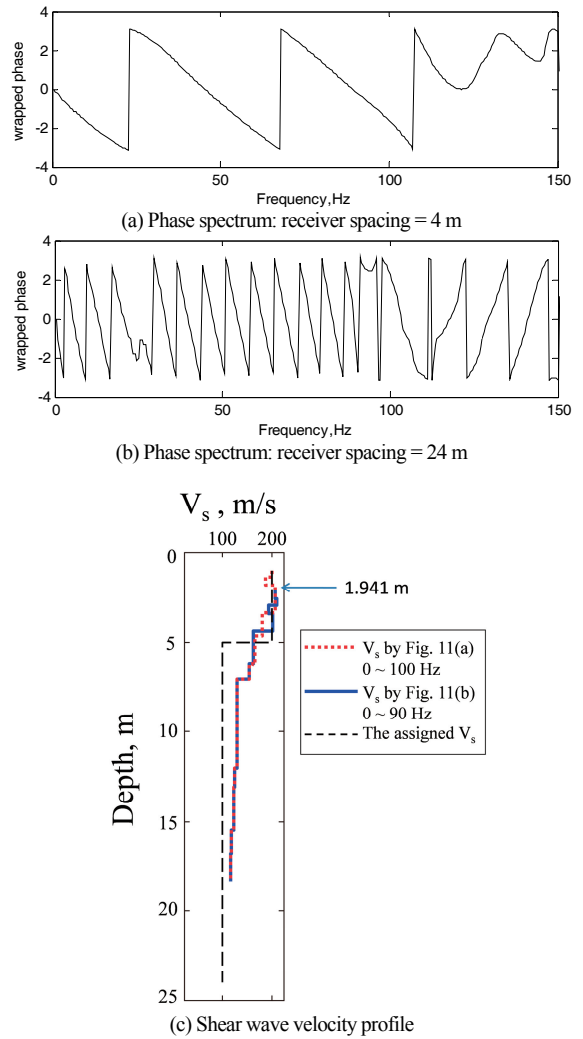

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m

(c) Shear wave velocity profile

Fig.10 Phase spectra and shear wave velocity profile obtained in Case 4

(a) Phase spectrum: receiver spacing = 4 m

(b) Phase spectrum: receiver spacing = 24 m

(c) Shear wave velocity profile

Fig.11 Phase spectra and shear wave velocity profile obtained in Case 5

quency corresponds to the quality of shear wave velocity obtained at the deeper soil. Therefore, the poorer quality could be due to the phenomenon of reflection and refraction waves produced at the interfaces of soil layers. Therefore, the more interfaces of soil layers there are, the more likely it is that the calculated shear wave velocity profile will be less accurate.

(2) Irregular profile

Case 5: Two layers (the depth of the upper layer and the lower layer are 5 and 35 m, and the shear wave velocities of soil are 200 and 100 m/s, respectively).

From Fig. 11, it is observed that for the 4 m receiver spacing case, the analyzable bandwidth is lower than 100 Hz. For the 24 m receiver spacing case, the analyzable bandwidth is lower than 90 Hz. To obtain the shear wave velocity corresponding to depth of 1 m, the demand bandwidth must be lower than 100 Hz for Case 5, the irregular profile.

(3) Summary of the effect of the soil profile

The discussions of the effect of the soil profile are summarized in Table 4. The demand bandwidth of cross power spectrum is affected mainly by shear wave velocity of the upper soil layer. Therefore, when applying the numerical model to simulate SASW testing, the demand bandwidth chosen should refer to shear wave velocity of the upper layer as an initial guess for numerical simulation of the SASW test.

7. Additional considerations

The associated factors for numerical stability (e.g. effect of grid size, time step) can be case dependent, depending on the numerical modeling and method of discretization. We used the analyzable bandwidth as the frequency range to reduce abnormal fluctuation in wrapped cross power spectrum. However, multiple modes may participate in certain frequency ranges to cause the cross power spectrum becoming not analyzable. This is also the limitation of cross power spectrum because it cannot reveal multiple modes in the plot of phase velocity versus frequency. The dispersion curves with multiple modes shall be analyzed using the method of Multi-channel Analysis of Surface Waves method. In addition, there exist analytical solutions for the dispersion curves in a soil media with multiple horizontal interfaces. They can be used to compare the results of the numerical simulation in this study. Also, the factor of groundwater was not considered in this study but it can be an important factor affecting the results of cross power spectrum.

4. Conclusion

This study performed a parametrical study for the numerical simulations of SASW tests. The influencing factors include shear wave velocity of soil, grid size of mesh, time step, Rayleigh damping ratio, impact source magnitude, and soil profile. The findings of this study are summarized as follows.

The finer the grid size of the mesh is, the larger the range of the analyzable bandwidth will be. We generally can reduce the occurrence of the abnormal phase fluctuation with smaller mesh size.

The analyzable bandwidth range becomes smaller when the damping of soil increases due to wave decay. It is more likely that the cross power spectrum will further deteriorate when the receiver spacing is also large.

The shear wave velocity of soil layer dominates the fluctuation of cross power spectrum and the demand bandwidth.

The demand bandwidth increases with increasing shear wave velocity of soil. The demand bandwidth can be estimated by the shear wave velocity of the topmost layer.

Acknowledgement

Table 4   Analyzable and demand bandwidth of the cross power spectrum

Soil profile		Analyzable bandwidth (Hz)		Demand bandwidth (Hz)
		Space = 4 m	Space = 24 m	
Regular profile	Case 1	0 ~ 100	0 ~ 50	0 ~ 50
	Case 2	0 ~ 70	0 ~ 40	0 ~ 50
	Case 3	0 ~ 100	0 ~ 70	0 ~ 100
	Case 4	0 ~ 100	0 ~ 50	0 ~ 50
Irregular profile	Case 5	0 ~ 100	0 ~ 90	0 ~ 100

References

[1] Chen, J.C., 2006, *Numerical Study on the Effects of Noise on the Dispersion Curve for Surface Wave*, Master thesis, Department of Civil Engineering, Feng Chia University, Taichung, Taiwan (in Chinese).

[2] Gucunski, N. and Woods, R.D., 1992, "Numerical Simulation of the SASW Test," *Soil Dynamics and Earthquake Engineering,* 11, 213-227.

[3] Hiltunen, D.R., and Woods, R.D., 1990, "Variables Affecting the Testing of Pavements by the Surface Wave Method," Transportation Research Record, 1260, 42-52.

[4] Heisey, J.S., Stokoe II, K.H., and Meyer, A.H., 1982, "Moduli of Pavement Systems from Spectral Analysis of Surface Waves," Transportation Research Record, 852, 22-31.

[5] Itasca Consulting Group, Inc., 2008, *Fast Lagrangian Analysis of Continua*, Version 6.0, User's Manual, Minneapolis, Minnesota, USA.

[6] Jiang, F.S., 2004, *Preliminary study of shear wave profile in soil layers by experimental dispersion curve*, Master thesis, Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan (in Chinese).

[7] Karray, M. and Lefebvre, G., 2009, "Techniques for Mode Separation in Rayleigh Wave Testing," *Soil Dynamics and Earthquake Engineering,* 29(4), 607-619.

[8] Kuhlemeyer, R.L., Lysmer, J., 1973, "Finite Element Method Accuracy for Wave Propagation Problem," *Journal of the Soil Mechanics and Foundations* Division, ASCE, 99(5), 421-427.

[9] Lin, C.P. and Lin, C.H., 2007, "Effect of lateral heterogeneity on surface wave testing: Numerical simulations and a countermeasure", *Soil Dynamics and Earthquake Engineering,* 27, 541-552.

[10] Lin, Y.T., 2010, *Numerical Simulations of Spectral Analysis of Surface Wave Method (SASW)*, Master thesis, Department of Soil and Water Conservation, National Chung Hsing University, Taichung, Taiwan (in Chinese).

[11] Lin, Y.T. and Feng, Z.Y., 2010, "Applying Hilbert-Huang Transform to the Spectral Analysis of Surface Wave Method", *Journal of Chinese Soil and Water Conservation,* Vol. 41, No. 2, 137-146 (in Chinese).

[12] Nazarian, S., Stokoe II, K.H., and Hudson, W.R., 1983, "Use of spectral analysis of surface waves method for determination of moduli and thickness of pavement systems", Transportation Research Records, 930, 38-45.

[13] Satoh, T., Poran, C.J., Yamagata, K. and Rodriquez, J.A., 1991, "Soil Profiling by Spectral Analysis of Surface Waves," Proc. of 2nd International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, St. Louis, Missouri, 1429-1434.

[14] Sheu, J.C., Stokoe II, K.H., and Roesset, J.M., 1988, "Effect of Reflected Waves in SASW Testing of Pavements," Transportation Research Record, 1196, 51-61.